EJERCICIOS DE MÁXIMOS Y MÍNIMOS

1. $\mathbf{f(x) = x^3 - 3x + 2}$

- $f'(x) = 3x^2 - 3 = 0$
- $f''(x) = 6x$
- $f''(-1) = -6$ Máximo
- $f''(1) = 6$ Mínimo
- $f(-1) = (-1)^3 - 3(-1) + 2 = 4$
- $f(1) = (1)^3 - 3(1) + 2 = 0$
- **Máximo(−1, 4)** **Mínimo(1, 0)**

2. $f(x) = 3x - x^3$

- $f'(x) = 3 - 3x^2$ $3 - 3x^2 = 0$
- $x = -1$ $x = 1$
- $f''(x) = -6x$ $f''(-1) > 0$ $f''(1) < 0$
- Mínimo $(-1, -2)$
- Máximo $(1, 2)$

x	$(-\infty,-1)$	$(-1,1)$	$(1,\infty)$
$f'(x)$	$-$	$+$	$-$
	↘	↗	↘

- **Creciente:**
- $(-1,1)$
- **Decreciente:**
- $(-\infty,-1)\cup(1,\infty)$

3. $f(x)=x^4-8x^2+3$

- $f'(x)=4x^3-16x$ $\quad\quad 4x^3-16x=0$
- $x=-2$ $\quad\quad x=0$ $\quad\quad x=2$
- $f''(x)=12x^2-16$
- $f''(-2)=12(-2)^2-16>0$ $\quad\quad$ Mínimo $(-2,-13)$
- $f''(0)=12(0)^2-16<0$ $\quad\quad$ Máximo $(0,3)$
- $f''(2)=12(2)^2-16>0$ $\quad\quad$ Mínimo $(2,-13)$

4. $f(x)=\dfrac{x^2-x-2}{x^2-6x+9}$

- $f(x)=\dfrac{x^2-x-2}{x^2-6x+9}=\dfrac{x^2-x-2}{(x-3)^2}$
- $f'(x)=\dfrac{(2x-1)(x-3)^2-(x^2-x-2)2(x-3)}{(x-3)^4}=\dfrac{-5x+7}{(x-3)^3}$
- $\dfrac{-5x+7}{(x-3)^3}=0 \qquad -5x+7=0 \qquad x=\dfrac{7}{5}$
- $f''(x)=\dfrac{10x-6}{(x-3)^4} \qquad f''\left(\dfrac{7}{5}\right)=\dfrac{10\left(\dfrac{7}{5}\right)-6}{\left(\left(\dfrac{7}{5}\right)-3\right)^4}>0$
- $f(x)=\dfrac{\left(\dfrac{7}{5}\right)^2-\left(\dfrac{7}{5}\right)-2}{\left(\dfrac{7}{5}\right)^2-6\left(\dfrac{7}{5}\right)+9}=-\dfrac{9}{16}$ *Mínimo* $\left(\dfrac{7}{5},-\dfrac{9}{16}\right)$

5. $f(x)=e^x\left(2x^2+x-8\right)$

- $f'(x)=e^x\left(2x^2+x-8\right)+e^x(4x+1)=e^x\left(2x^2+5x-7\right)$
- $e^x\left(2x^2+5x-7\right)=0 \qquad x=1 \qquad x=-\dfrac{7}{2}$
- $f''(x)=e^x\left(2x^2+9x-2\right)$
- $f''(1)=e^1\left(2\cdot 1^2+9\cdot 1-2\right)>0 \qquad f(1)=-5e$
- Mínimo $(1,-5e)$
- $f''\left(-\dfrac{7}{2}\right)=e^{-\frac{7}{2}}\left(2\cdot\left(-\dfrac{7}{2}\right)^2+9\cdot\left(-\dfrac{7}{2}\right)-2\right)<0 \qquad f\left(-\dfrac{7}{2}\right)=13e^{-\frac{7}{2}}$
- Máximo $\left(-\dfrac{7}{2},13e^{-\frac{7}{2}}\right)$

6. $f(x) = x + \ln\left(x^2 - 1\right)$

- $x^2 - 1 > 0 \qquad x^2 - 1 = 0 \qquad x = \pm 1$
x	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
	+	−	+
- $D = (-\infty, -1) \cup (1, \infty)$
- $f'(x) = 1 + \dfrac{2x}{x^2 - 1} = \dfrac{x^2 + 2x - 1}{x^2 - 1}$
- $\dfrac{x^2 + 2x - 1}{x^2 - 1} = 0 \qquad x = -1 + \sqrt{2} \notin D \qquad x = -1 - \sqrt{2}$
- $f''(x) = \dfrac{-2\left(x^2 + 1\right)}{\left(x^2 - 1\right)^2} \qquad f''(-1 - \sqrt{2}) = \dfrac{-2\left((-1 - \sqrt{2})^2 + 1\right)}{\left((-1 - \sqrt{2})^2 - 1\right)^2} < 0$
- En $x = -1 - \sqrt{2}$ hay un máximo

7. $f(x) = \text{sen } 2x$

- $f'(x) = 2\cos 2x$ $\quad 2\cos 2x = 0$
- $2x = \frac{\pi}{2} + 2\pi k$ $\quad x_1 = \frac{\pi}{4} + k\pi$
- $2x = \frac{3\pi}{2} + 2\pi k$ $\quad x_1 = \frac{3\pi}{4} + k\pi$
- $f''(x) = -4\,\text{sen } 2x$
- $f''\left(\frac{\pi}{4}\right) = -4\,\text{sen } 2\left(\frac{\pi}{4}\right) < 0$ $\quad f\left(\frac{\pi}{4}\right) = \text{sen } 2\left(\frac{\pi}{4}\right) = 1$
- Máximo $\left(\frac{\pi}{4} + k\pi, 1\right)$
- $f''\left(\frac{3\pi}{4}\right) = -4\,\text{sen } 2\left(\frac{3\pi}{4}\right) > 0$ $\quad f\left(\frac{3\pi}{4}\right) = \text{sen } 2\left(\frac{3\pi}{4}\right) = 1$
- Mínimo $\left(\frac{3\pi}{4} + k\pi, -1\right)$